# EPSTEIN INSTITUTE SEMINAR ▪ ISE 651

## Convergent Decentralized Algorithms for Certain Nash Equilibrium Problems


**Andrew Liu**
Assistant Professor
School of Industrial Engineering
Purdue University



**ABSTRACT –** While algorithms for computing a Nash equilibrium of noncooperative games have been developed over decades, from fixed point-based to Newton-based algorithms, most of such algorithms only focus on a game's equilibrium state and are incapable of explaining how such a state is reached (by the self-interested players in the game). Such a limitation may not be an issue for games of a unique equilibrium; for games of multiple equilibria, however, just computing a particular equilibrium (without further justifications) may be of limited use for understanding players' strategies. Though theories of behavioral and experimental game theory have been developed to study how a specific equilibrium is reached, we would like to focus on the middle ground; namely, to use decentralized, algorithmic approaches (such as best-response-dynamics, fictitious play, etc) to study equilibrium formation. Such approaches are also motivated by a different reason; that is, finding an equilibrium of certain games can be shown to be equivalent of solving a single optimization problem (such as potential games). Hence the decentralized approaches can provide an algorithmic framework amenable for high scalability computing of large-scale optimization problems.

It is well-known that a decentralized approach may not lead to a Nash equilibrium of a generic game. Hence in this work we focus on two special classes of games – multiagent network interdiction games and potential games under exogenous uncertainty – to study the convergent properties of best-response-type algorithms. The former class models the interactions among multiple interdictors facing different adversaries who operate on a common network. We start with the decentralized shortest path interdiction (DSPI) game, and present results for the existence and uniqueness of equilibria. Regularized best-response algorithms applied to the DSPI game are then presented with numerical results. The decentralized algorithms are then used to empirically analyze the average efficiency loss (termed as average-case price of anarchy) associated with the equilibria of DSPI games, with such loss caused by the lack of coordination among noncooperative interdictors.

In the second part, we analyze decentralized approaches to compute a Nash equilibrium under exogenous uncertainty for potential games, which have wide ranging practical applications. Under the assumption that players are risk neutral, we prove the convergence of decentralized sampling based approximation schemes to an equilibrium. The algorithms are then applied to two relevant applications – spatial Nash-Cournot models in an electric power market and stochastic network traffic routing problems.


**SPEAKER BIO –Dr. Andrew (Lu)** Liu joined the School of Industrial Engineering at Purdue University as an Assistant Professor in August 2009. He received his Ph.D. in Applied Mathematics and Statistics from The Johns Hopkins University in 2009. He also holds an M.S. in Mathematical Sciences from Johns Hopkins, and a B.S. in Applied Mathematics from Beijing Institute of Technology. Dr. Liu's primary research interests lie in the interactions of optimization, game theory and industrial organization, with applications to model and analyze energy markets, environmental policies, and smart grid. He is the Purdue Discovery Park Research Fellow in 2014 – 2015, and his work has been funded by NSF, DOE and U. S. Air Force. In addition to his academic experience, Dr. Liu has worked at ICF International as a Senior Associate, responsible for developing optimization and stochastic models on investment in energy markets and environmental policy analysis.

USC Viterbi
School of Engineering
*Daniel J. Epstein Department of Industrial and Systems Engineering*

**TUESDAY, SEPTEMBER 15, 2015**
3:30PM – 4:50PM
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